Sum Labeling for Arbitrary Supersubdivision of Comb, $P_n \odot 2K_1$ & Spider

- Gerard Rozario J* - Jon Arockiaraj J**

Abstract

A *sum labeling* is a mapping from the vertices of G into the positive integers such that, for any two vertices u, v V (G) with labels (u) and (v), respectively, (uv) is an edge iff (u) + (v) is the label of another vertex in V (G). Any graph supporting such a labeling is called a *sum graph*. It is necessary to add (as a disjoint union) a component to sum label a graph. This disconnected component is a set of isolated vertices known as *isolates* and the labeling scheme that requires the fewest isolates is termed *optimal*. The number of isolates required for a graph to support a sum labeling is known as the *sum number* of the graph.

In this paper, we will obtain *optimal sum labeling scheme* for arbitrary super subdivision of comb, and spider

Keywords: Sum Labeling, Sum Number, Sum Graph, Path Union, Arbitrary Supersubdivision
2010 AMS Subject Classification: 05C78

Introduction

All the graphs considered here are simple, finite and undirected. For all terminologies and notations we follow Harary [5] and graph labeling as in [2]. Sum labeling of graphs was introduced by Harary [6] in 1990. Following definitions are useful for the present study.

Definition 1.1 A *Sum Labeling* is a mapping from the vertices of G into the positive integers such that, for any two vertices u, v V (G) with labels (u) and (v), respectively, (uv) is an edge iff (u) + (v) is the label of another vertex in V (G). Any graph supporting such a labeling is called a *Sum Graph*.

Definition 1.2 It is necessary to add (as a disjoint union) a component to sum label a graph. This disconnected component is a set of isolated vertices known as *Isolates* and the labeling scheme that requires the fewest isolates is termed *Optimal*.

Definition 1.3 The number of isolates required for a graph G to support a sum labeling is known as the *Sum Number* of the graph. It is denoted as.

Definition 1.4 [8] Let G be a graph with q edges. A graph H is called a *Super subdivision* of G if H is obtained from G by replacing every edge e_i of G by a complete bipartite graph $K_{2\ m_i}$ for some m_i, *1 ″ i ″ q* in such a way that the end vertices of each e_i are identified with the two vertices of 2-vertices part of $K_{2\ m_i}$ after removing the edge e_i from graph G. If m_i is varying arbitrarily for each edge e_i then super subdivision is called arbitrary super subdivision of G.

Definition 1.5 The graph obtained by adding a pendent edge to each vertex of a path of n vertices is called a *Comb*. It is denoted by.

Definition 1.6 is the graph obtained by adding two pendent edges to each vertex of a path of n vertices.

*PG and Research Department of Mathematics, St. Joseph's College of Arts and Science, Cuddalore, Tamilnadu, India.
rozario.gerard@yahoo.com
**PG and Research Department of Mathematics, St. Joseph's College of Arts and Science, Cuddalore, Tamilnadu, India.
rozario. jonarockiaraj@gmail.com

Definition 1.7 (Chung et. al [1]) A tree is called a *spider* if it has a center vertex c of degree k > 1 and each other vertex either is a leaf (pendent vertex) or has degree 2. Thus a spider is an amalgamation of k paths with various lengths. If it has x_1 paths of length a_1, x_2 paths of length a_2, ..., x_n paths of length a_m, we denote the spider by SP($a_1^{x_1}, a_2^{x_2}, \ldots, a_m^{x_n}$) where $a_1 < a_2 < \ldots < a_m$ and $x_1 + x_2 + \ldots + x_n = k$

Optimal Sum Labeling Scheme for Arbitrary Super Subdivision of Comb, & Spider

Sethuraman et.al [8], introduced a new method of construction called Supersubdivision of graph and proved that arbitrary supersubdivision of any path and cycle C_n are graceful. Kathiresan et.al [7], proved that arbitrary supersubdivision of any star is graceful. In [3], Gerard Rozario et.al proved that arbitrary super subdivision of path, cycle and star are sum graph with sum number 2. In [4], Gerard Rozario et.al proved that arbitrary super subdivision of crown, armed crown and t-thorny ring are sum graph with sum number 2.

In this section, we prove that graphs obtained by arbitrary super subdivision of comb, and spider are sum graph with sum number 2.

Theorem: 2.1

Arbitrary supersubdivision of a comb is sum graph with

Proof: Let G be a comb. Let v_i *(1 ″ i ″ n)* be the vertices of path and w_i be the pendent vertex adjacent to v_i *(1 ″ i ″ n)*. Let H be the arbitrary supersubdivision of G which is obtained by replacing every edge of G with. Let. Let u_j be the vertices which are used for arbitrary supersubdivision of G where. Let x and y be two isolated vertices. Therefore, the vertex set of H is V(H) = { $v_1, v_2, \ldots, v_n, w_1, w_2, \ldots w_n, u_1, u_2, \ldots\ldots, u_m$ }. Define f : V(H) → N

$$f(v_1) = 1 \qquad f(w_1) = 2$$
$$f(v_i) = f(v_{(i-1)}) + 2 \ \ for\ 2 \le i \le n$$
$$f(w_i) = f(w_{(i-1)}) + 2 \ \ for\ 2 \le i \le n$$
$$f(u_1) = m + n$$
$$f(u_j) = f(u_{j-1}) - 1 \ \ for\ \ 2 \le j \le m$$
$$f(x) = f(u_1) + 1 \text{ and } f(y) = f(u_1) + 2$$

Thus, arbitrary supersubdivision of comb is sum graph with sum number 2.

Illustration: *Sum labeling for arbitrary supersubdivision of comb* $P_4 \odot K_1$ *is shown in figure 2.1*

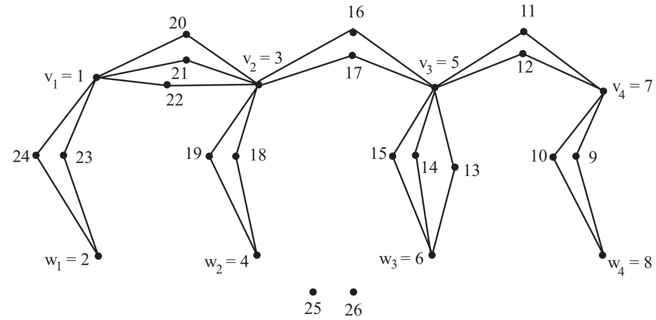

Figure 2.1

Theorem: 2.2 Arbitrary supersubdivision of $P_n \odot 2K_1$ is sum graph with $\sigma(G) = 2$.

Proof: Let G be the graph $P_n \odot 2K_1$. Let p_i *(1 ≤ i ≤ n)* be the vertices of path and p_{i1} and p_{i2} be the pendent vertices adjacent to p_i *(1 ≤ i ≤ n)*. G has 3n vertices and (3n – 1) edges. Let H be the arbitrary supersubdivision of G which is obtained by replacing every edge of G with $K_{2,mi}$. Let $m = \sum_1^{(3n-1)} m_i$. Let u_j be the vertices which are used for arbitrary supersubdivision of G where 1 ≤ i ≤ m. Let x and y be two isolated vertices. Therefore, the vertex set of H is V(H) = { $p_1, p_2, \ldots, p_n, p_{11}, p_{12}, \ldots p_{n1}, p_{n2}, u_1, u_2, \ldots\ldots, u_m$ }.

Define f : V(H) → N

$$f(p_1) = 1; \ \ f(p_{11}) = 2; \ \ f(p_{12}) = 3$$
$$for\ 2 \le i \le n$$
$$\begin{cases} f(p_i) = f(p_{i-1}) + 3 \\ f(p_{i1}) = f(p_{(i-1)1}) + 3 \\ f(p_{i2}) = f(p_{(i-1)2}) + 3 \end{cases}$$
$$f(u_1) = 3n + m$$
$$f(u_j) = f(u_j - 1) - \text{ for } 2 \le j \le m$$
$$f(x) = f(u_2) + 1 \text{ and } f(y) = f(u_2) + 2$$

Thus, arbitrary supersubdivision of $P_n \odot 2K_1$ is sum graph with sum number 2.

Illustration: *Sum labeling for arbitrary supersubdivision of* $P_4 \odot 2K_1$ *is given in figure 2.2*

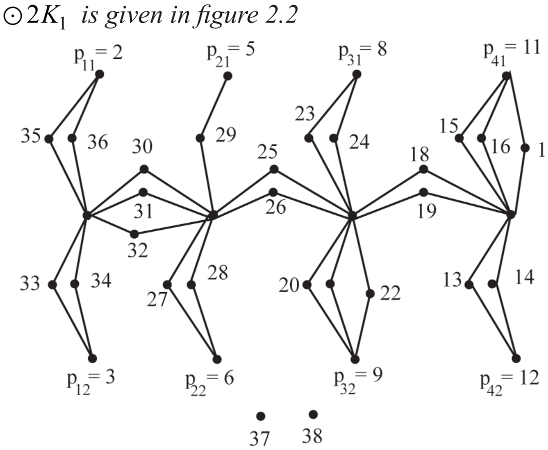

Figure 2.2

Theorem: 2.3 Arbitrary supersubdivision of the spider SP_n, is sum graph with $\sigma(G) = 2$.

Proof: Let G = SP_n, where SP_n is a spider of n vertices. G has n vertices and n-1 edges. Let the vertices of SP_n be $v_1, v_2, \ldots, v_n$. Let H be the arbitrary supersubdivision of G which is obtained by replacing every edge of G with $K_{2,mi}$. Let $m = \sum_1^{(n-1)} m_i$. Let u_j be the vertices which are used for arbitrary supersubdivision of G where $1 \leq j \leq m$. Let x and y be two isolated vertices. Therefore, the vertex set of H is $V(H) = \{ v_1, v_2, \ldots, v_n, u_1, u_2, \ldots\ldots, u_m \}$.

Define f : V(H) → N

$$f(v_i) = i \ \ for\ 1 \leq i \leq n$$

$$f(u_1) = n + m$$

$$f(u_i)\ f\ (u_{(i-1)}) + 1 \ \ for\ 2 \leq i \leq m$$

$$f(x) = f(u_1) + 1 \text{ and } f(y) = f(u_1) + 2$$

Thus, arbitrary supersubdivision of spider SP_n, is sum graph with sum number 2.

Illustration: *Sum labeling for arbitrary supersubdivision of spider is shown in figure 2.3*

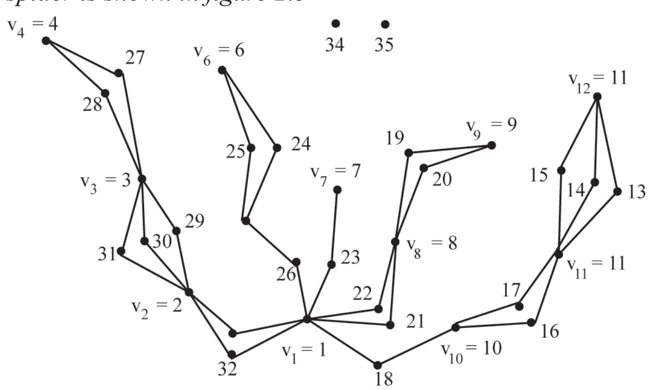

Figure 2.3

Reference

1. Chung P T and Lee S M (2008), *On the super edge-graceful spiders of even orders*, Journal of Combinatorial Mathematics and Combinatorial Computing, 64, 3-17.
2. Gallian J A (2009), *A dynamic survey of graph labeling*, The Electronics Journal of Combinatorics, 16, DS6.
3. Gerard Rozario J and Jon Arockiaraj J (**2012)**, *Sum Labeling for some cycle related graphs*; paper presented in National Seminar (UGC Sponsored) on Recent Trends in Distance in Graphs, **ANJAC, Sivakasi,** March 12 – 13.
4. Gerard Rozario J and Jon Arockiaraj J (2012), Sum Labeling for Crown, Armed Crown and t – thorny rings, *National Conference on Graph Theory Generalizations and Applications*, SASTRA University, Thanjavur, August 20 – 21.Harary F (1972), *Graph theory*, Addison Wesley, Reading, Massachusetts.
5. Harary F, Graph theory, Addison Wesley, Reading, Massachusetts, 1972.
6. Harary F (1990), *Sum graphs and Difference graphs*, Congress Numerantium, no.72, 101-108.
7. Kathiresan K. M, Amutha S (2004), "*Arbitrary supersubdivisions of stars are graceful*", Indian Journal of pure and applied Mathematics. 35(1), pp. 81-84.
8. Sethuraman G, Selvaraju P (2001), "*Gracefulness of arbitrary supersubdivisions of graphs*", Indian Journal of pure and applied Mathematics, 32(7), pp. 1059-1064.